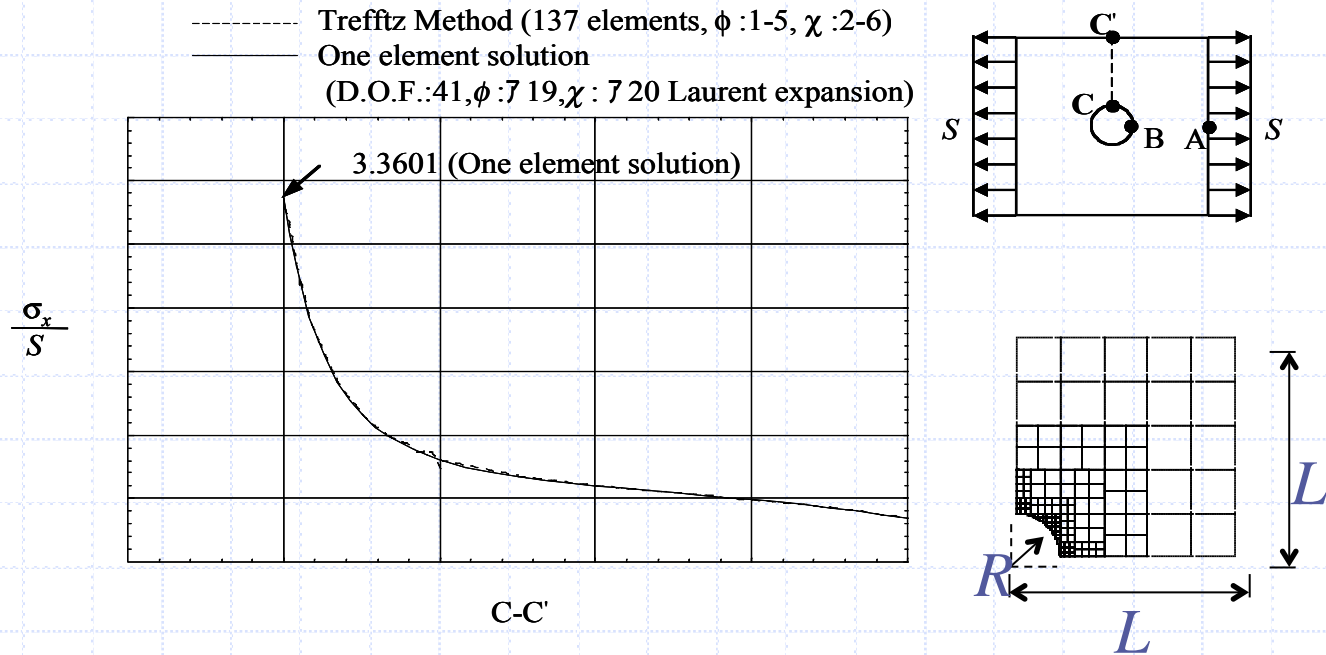
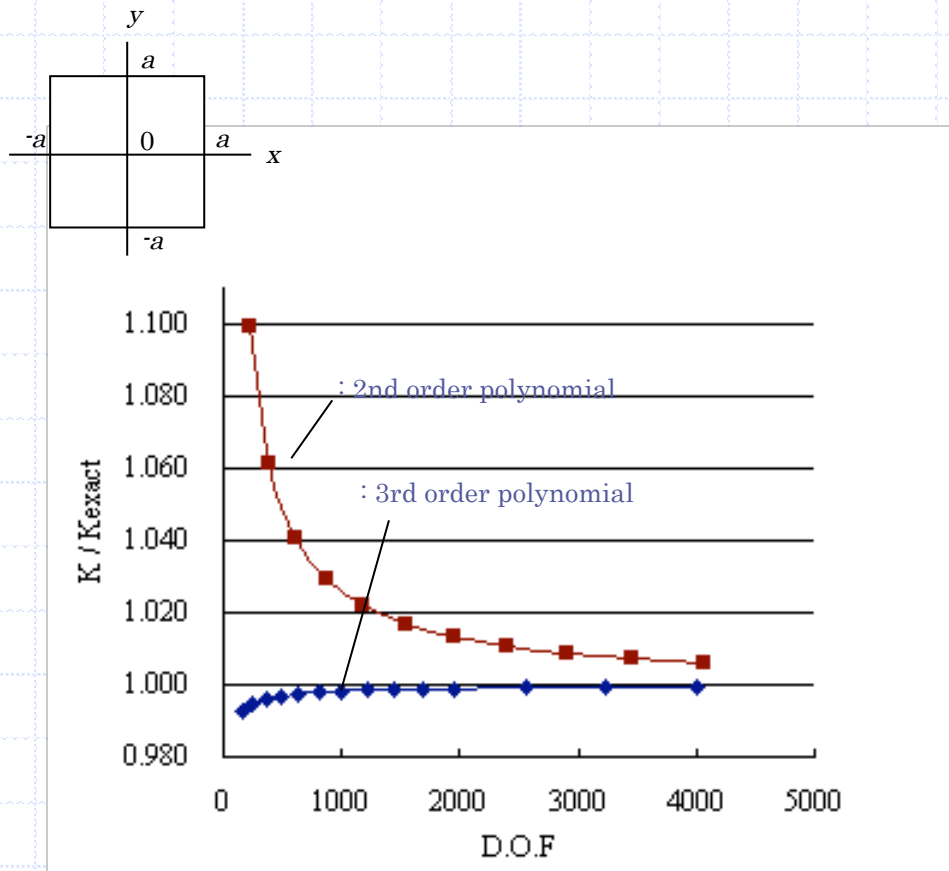


Stress distribution on section C-C' of a perforated square plate under uniaxial uniform loading



$R = 10\text{mm}, L = 50\text{mm}, S = 100\text{kgf/mm},$
 $E = 20000\text{kgf/mm}^2, \nu = 0.3$

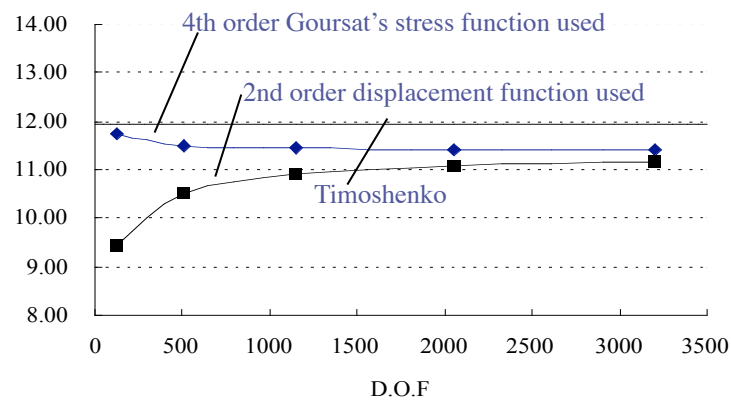
Analysis of torsional rigidity of an elastic bar with the square cross section (divided by square mesh)



| 6DOF / element | | 10 DOF / element | |
|-----------------------------|----------------------|------------------|----------------------|
| Mesh Div. | 2nd order polynomial | Mesh Div. | 3rd order polynomial |
| 2x 2 | 0.17708 | 3x 3 | 0.13951 |
| 4x 4 | 0.16498 | 4 x4 | 0.13961 |
| 6x 6 | 0.15456 | 5x 5 | 0.13984 |
| 8x 8 | 0.14921 | 6x 6 | 0.14002 |
| 10x 10 | 0.14635 | 7x 7 | 0.14014 |
| 12 x12 | 0.14469 | 8x 8 | 0.14023 |
| 14x 14 | 0.14365 | 9x 9 | 0.1403 |
| 16x 16 | 0.14295 | 10x 10 | 0.14035 |
| 18 x18 | 0.14247 | 11x 11 | 0.14039 |
| 20x 20 | 0.14211 | 12x 12 | 0.14041 |
| 22x 22 | 0.14185 | 13x 13 | 0.14044 |
| Timoshenko $K=0.1406(2a)^4$ | | | |

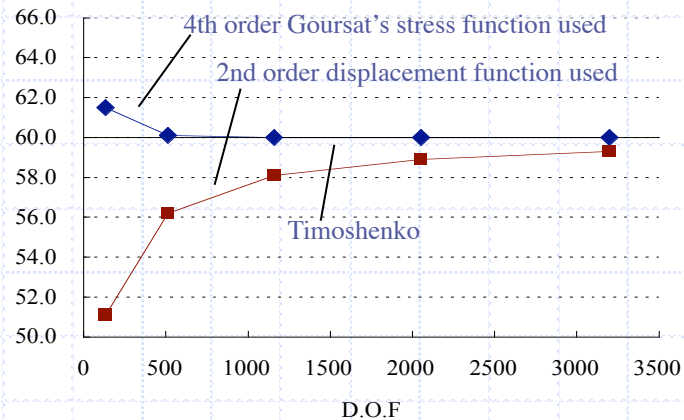
Inplane bending analysis of a cantilever plate subjected to a boundary shear of parabolic distribution (divided by square mesh)

V_A : vertical displacement at the point A



| Mesh Div. x NDOF | stress function used | displacement function used |
|------------------|----------------------|----------------------------|
| 4x 2x 16 | 11.7195 | 9.4399 |
| 8x 4 x16 | 11.4996 | 10.5163 |
| 12x 6 x16 | 11.4347 | 10.9196 |
| 16 x8x 16 | 11.4063 | 11.0912 |
| 20x 10 x16 | 11.3909 | 11.178 |

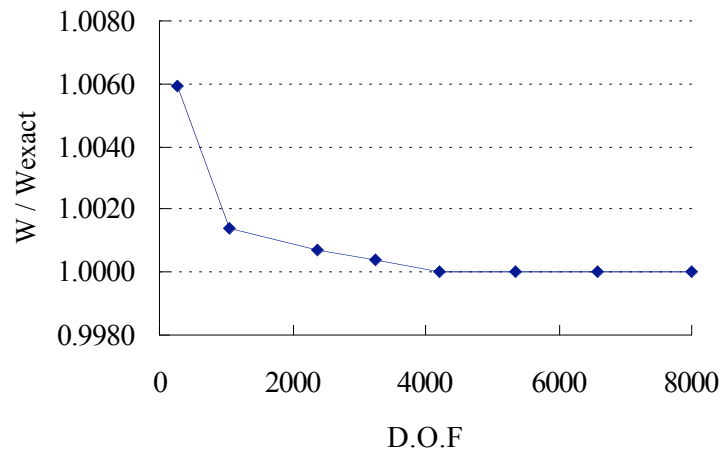
$(\sigma_x)_B$: stress at the point B



| Mesh Div. x NDOF | stress function used | displacement function used |
|------------------|----------------------|----------------------------|
| 4x 2x 16 | 61.4766 | 51.0777 |
| 8x 4 x16 | 60.0641 | 56.1607 |
| 12x 6 x16 | 60.0287 | 58.1254 |
| 16 x8x 16 | 60.0138 | 58.8946 |
| 20x 10 x16 | 60.0071 | 59.2698 |

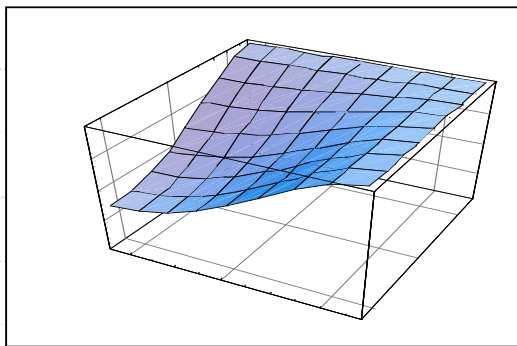
Finite element bending analysis of a square plate under uniformly distributed load using the newly proposed variational method.

Nonequilibrium 10th order polynomials of (x,y) were used for analysis.



Central deflection $w(0,0)$

| Mesh Div NDOF | $w(0,0)$ | w/w_{exact} |
|------------------|----------|---------------|
| 2 2 66 | 1.0059 | 1.27247 |
| 3 3 66 | 1.0014 | 1.26676 |
| 4 4 66 | 1.0014 | 1.26675 |
| 5 5 66 | 1.0006 | 1.26576 |
| 6 6 66 | 1.0007 | 1.26587 |
| 7 7 66 | 1.0004 | 1.26555 |
| 8 8 66 | 1.0000 | 1.26524 |
| 9 9 66 | 1.0000 | 1.26502 |
| 10 10 66 | 1.0000 | 1.26531 |
| 11 11 66 | 1.0000 | 1.26531 |



deflection (Mesh Div. NDOF = 8866)

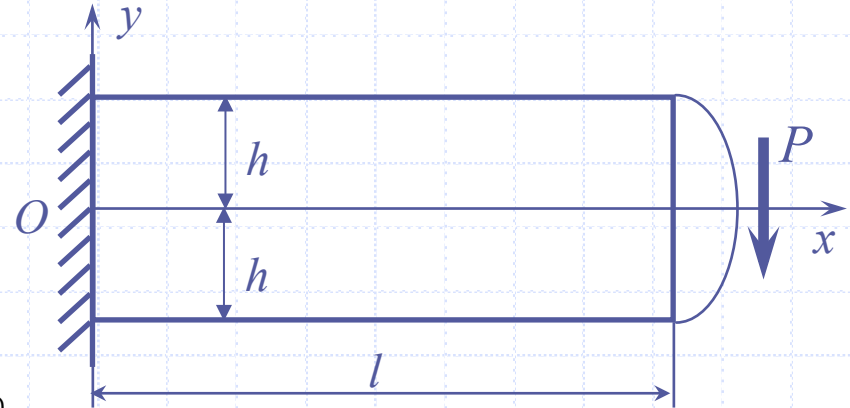
先端に垂直せん断力を受ける片持矩形板の面内曲げ問題のTK解析

平衡条件式:

$$\left. \begin{aligned} \nabla^2 u + \frac{(1+\nu)}{2} \frac{\partial}{\partial x} \left(-\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + f_x &= 0 \\ \nabla^2 v + \frac{(1+\nu)}{2} \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) + f_y &= 0 \end{aligned} \right\} \quad (1)$$

境界条件:

$$\left. \begin{aligned} x=0 : \text{固定端} \quad u(0, y) = v(0, y) &= 0 \\ y = \pm h : \text{自由端} \quad \tau_{xy}(x, \pm h) = 0, \sigma_y(x, \pm h) &= 0 \\ x=l : \text{荷重端} \quad \sigma_y(l, y) = 0, \tau_{xy} &= -\frac{P}{2I}(h^2 - y^2) \end{aligned} \right\} \quad (2)$$

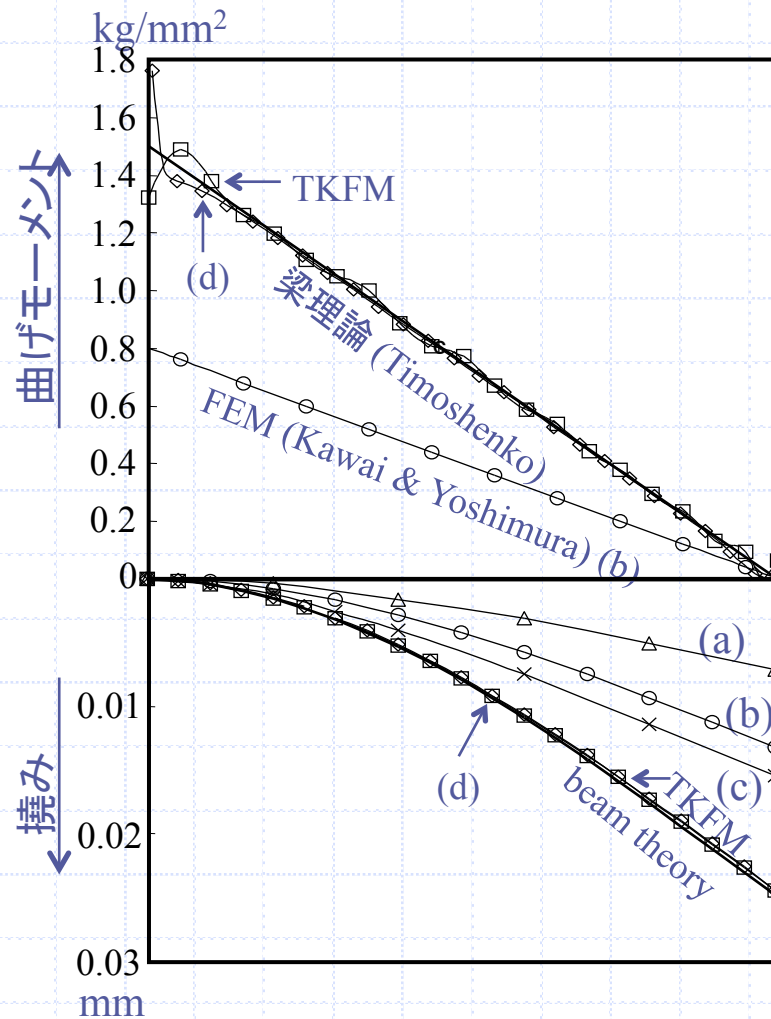
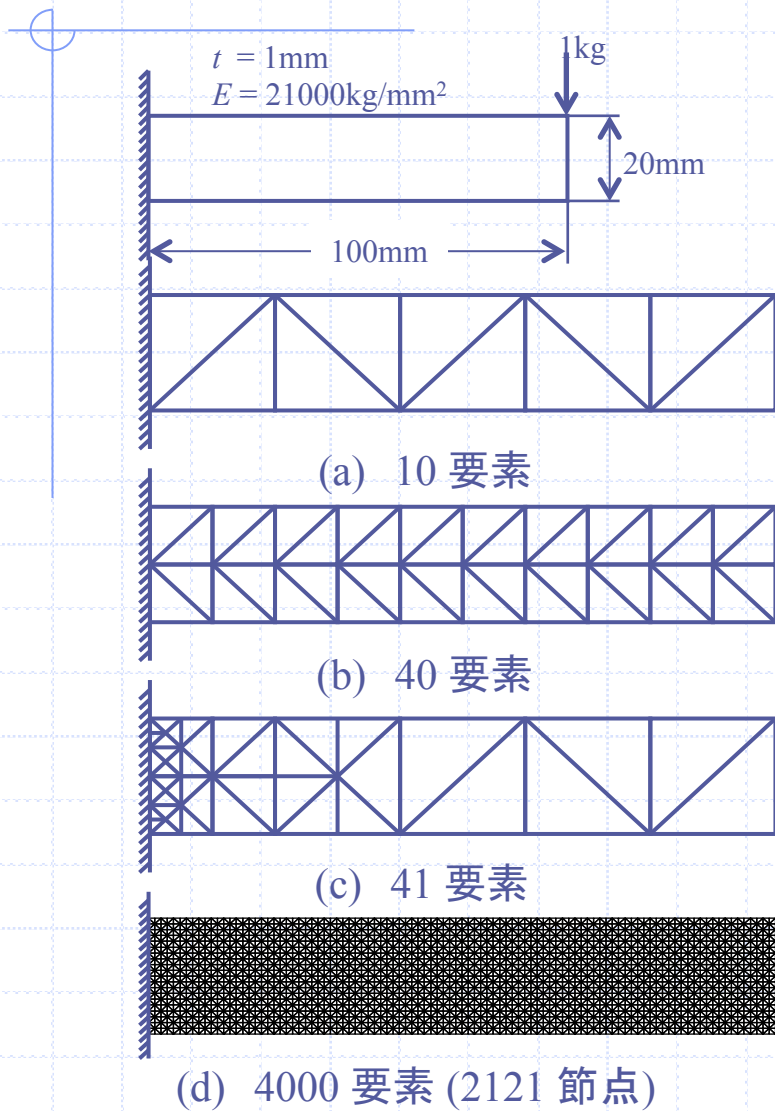


近似固定条件に対するTimoshneko解:

$$u(0,0) = v(0,0) = \frac{\partial v}{\partial x}(0,0) = 0 \quad (3)$$

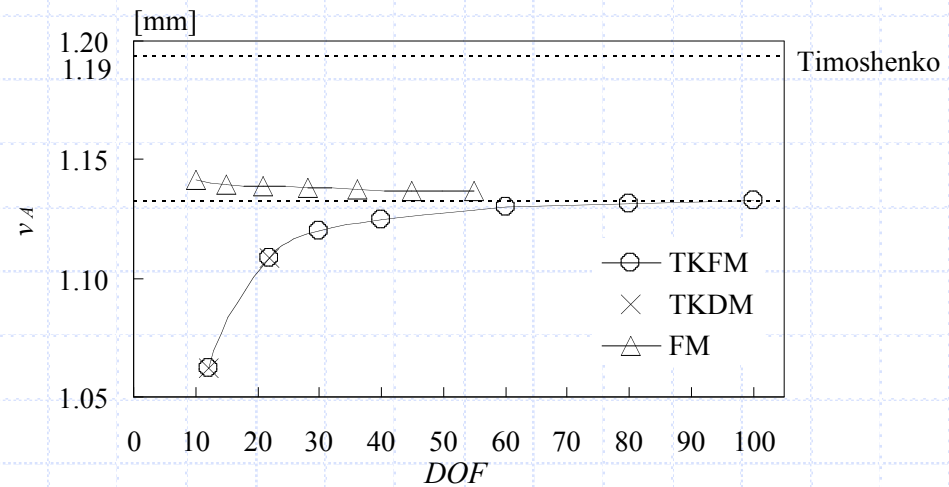
$$\left. \begin{aligned} u(x, y) &= -\frac{Px^2 y}{2EI} - \frac{\nu Py^2}{6EI} + \frac{Py^3}{6GI} + \left(\frac{Pl^2}{2EI} - \frac{Ph^2}{2GI} \right) y \\ v(x, y) &= \frac{\nu Pxy^2}{2EI} + \frac{Px^3}{6EI} - \frac{Pl^2 x}{2EI} + \frac{Pl^3}{3EI} \end{aligned} \right\} \quad (4)$$

CST要素を用いた有限要素解とTimoshenko解の比較

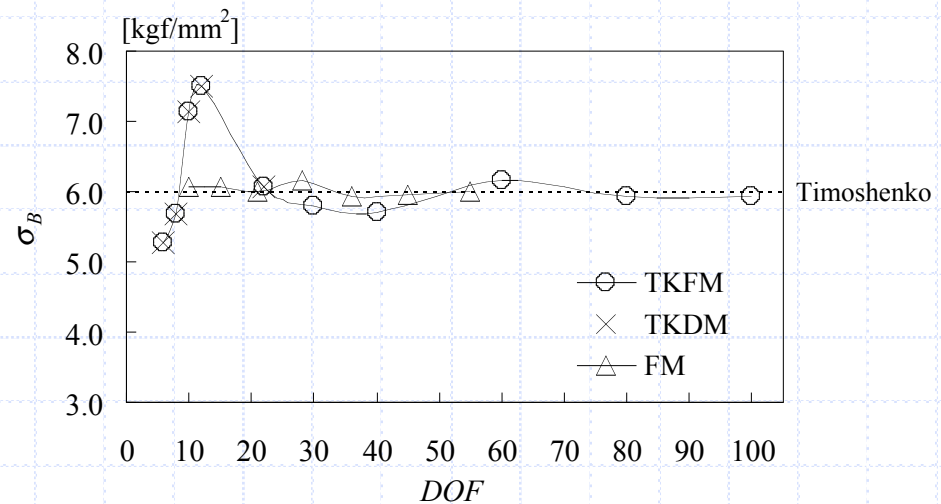


平衡法 (FM) 解と100項近似のTKFMとTKDM解の比較

| The deflection of point A [mm] | | | | |
|--------------------------------|--------|--------|-----|--------|
| DOF | TKFM | TKDM | DOF | FM |
| 6 | 0.7377 | 0.7377 | 10 | 1.1417 |
| 8 | 0.7812 | 0.7812 | 15 | 1.1398 |
| 10 | 0.9815 | 0.9815 | 21 | 1.1387 |
| 12 | 1.0621 | 1.0621 | 28 | 1.1379 |
| 22 | 1.1088 | 1.1088 | 36 | 1.1373 |
| 30 | 1.1202 | | 45 | 1.1369 |
| 40 | 1.1247 | | 55 | 1.1365 |
| 60 | 1.1302 | | | |
| 80 | 1.1312 | | | |
| 100 | 1.1324 | | | |

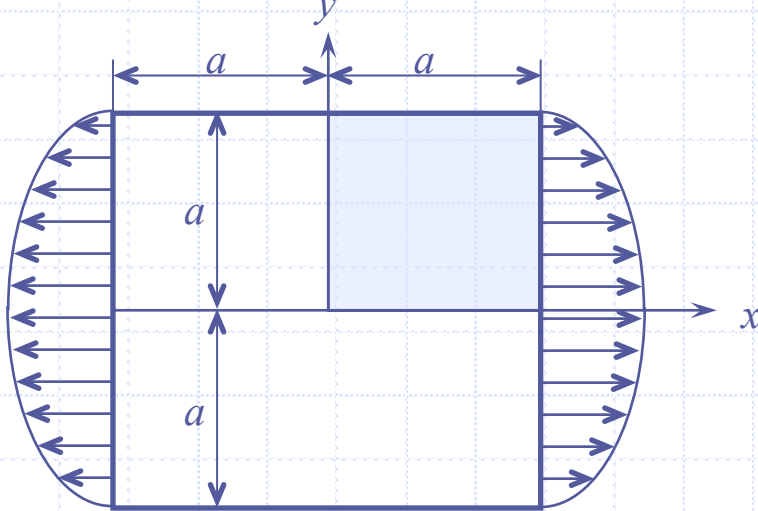


| The bending stress at point B σ_x [kgf/mm ²] | | | | |
|---|--------|--------|-----|--------|
| DOF | TKFM | TKDM | DOF | FM |
| 6 | 5.2647 | 5.2647 | 10 | 6.0781 |
| 8 | 5.6759 | 5.6759 | 15 | 6.0657 |
| 10 | 7.1358 | 7.1358 | 21 | 6.0113 |
| 12 | 7.4969 | 7.4969 | 28 | 6.1516 |
| 22 | 6.0718 | 6.0718 | 36 | 5.9345 |
| 30 | 5.7907 | | 45 | 5.9479 |
| 40 | 5.7008 | | 55 | 5.9899 |
| 60 | 6.1502 | | | |
| 80 | 5.9393 | | | |
| 100 | 5.9286 | | | |

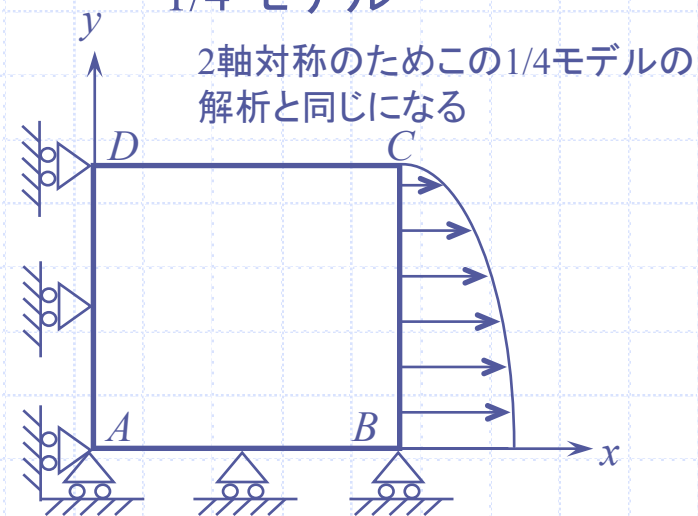


一方向に放物線状分布引張荷重を受ける 矩形板の面内変形解析

解析モデル



1/4 モデル



境界条件

$$\left. \begin{aligned} x = \pm a: \quad \tau_{xy} = 0, \quad \sigma_x = S \left(1 - \frac{y^2}{a^2} \right) \\ y = \pm a: \quad \tau_{xy} = 0, \quad \sigma_y = 0 \end{aligned} \right\}$$

寸法及び材料定数

$$a = 100\text{mm}$$

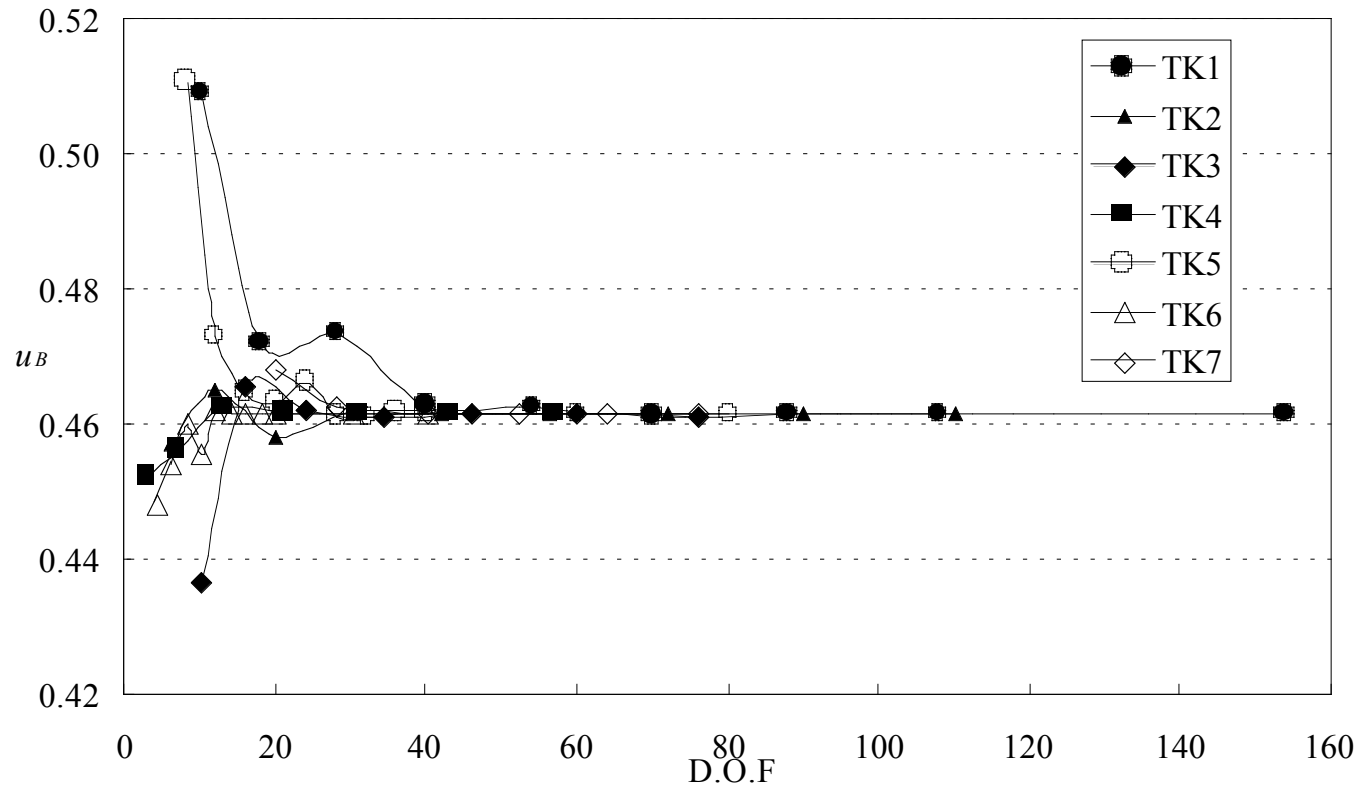
$$S = 100$$

$$E = 20000\text{kgf} / \text{mm}^2$$

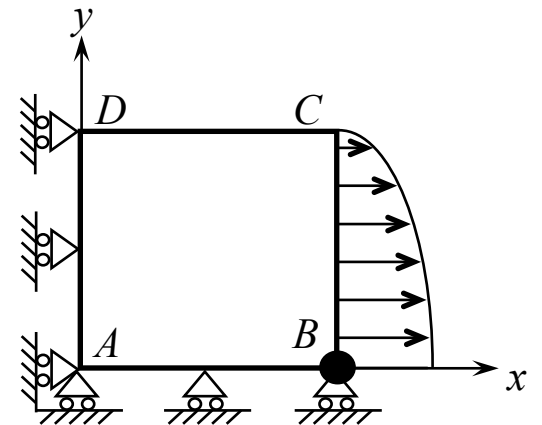
$$\nu = 0.3$$

解析解的手法（解法8）を除いた7つの解法の解の比較

| DOF | TK1 | DOF | TK2 | DOF | TK3 | DOF | TK4 | DOF | TK5 | DOF | TK6 | DOF | TK7 |
|-----|---------|-----|---------|-----|---------|-----|---------|-----|---------|-----|---------|-----|---------|
| 10 | 0.50918 | 6 | 0.45747 | 10 | 0.43648 | 3 | 0.45237 | 8 | 0.51110 | 4 | 0.44828 | 20 | 0.46833 |
| 18 | 0.47225 | 12 | 0.46539 | 16 | 0.46598 | 7 | 0.45639 | 12 | 0.47319 | 6 | 0.45434 | 28 | 0.46293 |
| 28 | 0.47352 | 20 | 0.45823 | 24 | 0.46230 | 13 | 0.46267 | 16 | 0.46482 | 8 | 0.46006 | 40 | 0.46196 |
| 40 | 0.46285 | 30 | 0.46220 | 34 | 0.46112 | 21 | 0.46181 | 20 | 0.46348 | 10 | 0.45551 | 52 | 0.46166 |
| 54 | 0.46271 | 42 | 0.46192 | 46 | 0.46192 | 31 | 0.46163 | 24 | 0.46641 | 12 | 0.46238 | 64 | 0.46157 |
| 70 | 0.46145 | 56 | 0.46165 | 60 | 0.46181 | 43 | 0.46167 | 28 | 0.46135 | 14 | 0.46186 | 76 | 0.46156 |
| 88 | 0.46153 | 72 | 0.46168 | 76 | 0.46131 | 57 | 0.46168 | 32 | 0.46101 | 16 | 0.46177 | | |
| 108 | 0.46157 | 90 | 0.46168 | | | | | 36 | 0.46174 | 18 | 0.46163 | | |
| 154 | 0.46171 | 110 | 0.46168 | | | | | 40 | 0.46179 | 20 | 0.46178 | | |
| | | | | | | | | 60 | 0.46170 | 30 | 0.46170 | | |
| | | | | | | | | 80 | 0.46169 | 40 | 0.46178 | | |

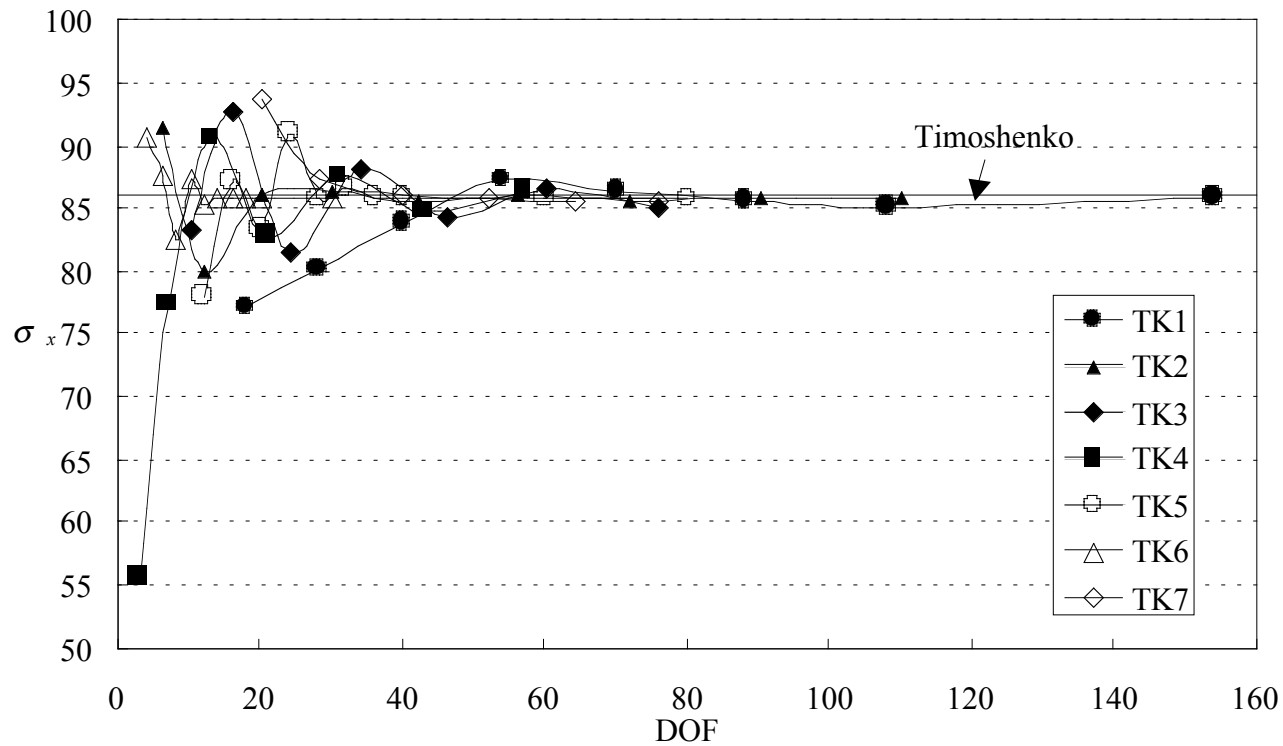


B点の水平変位 u_B
の計算値の比較

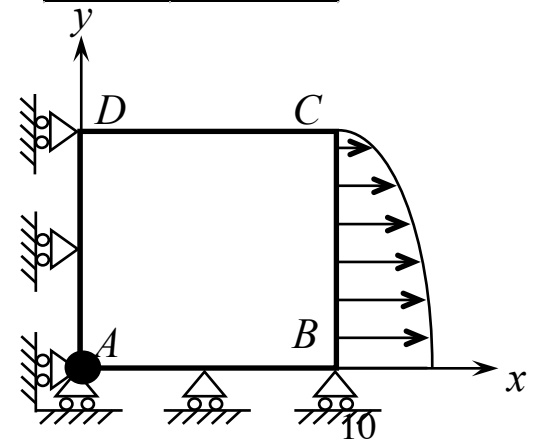


A点の引張応力 σ_x の計算値の比較

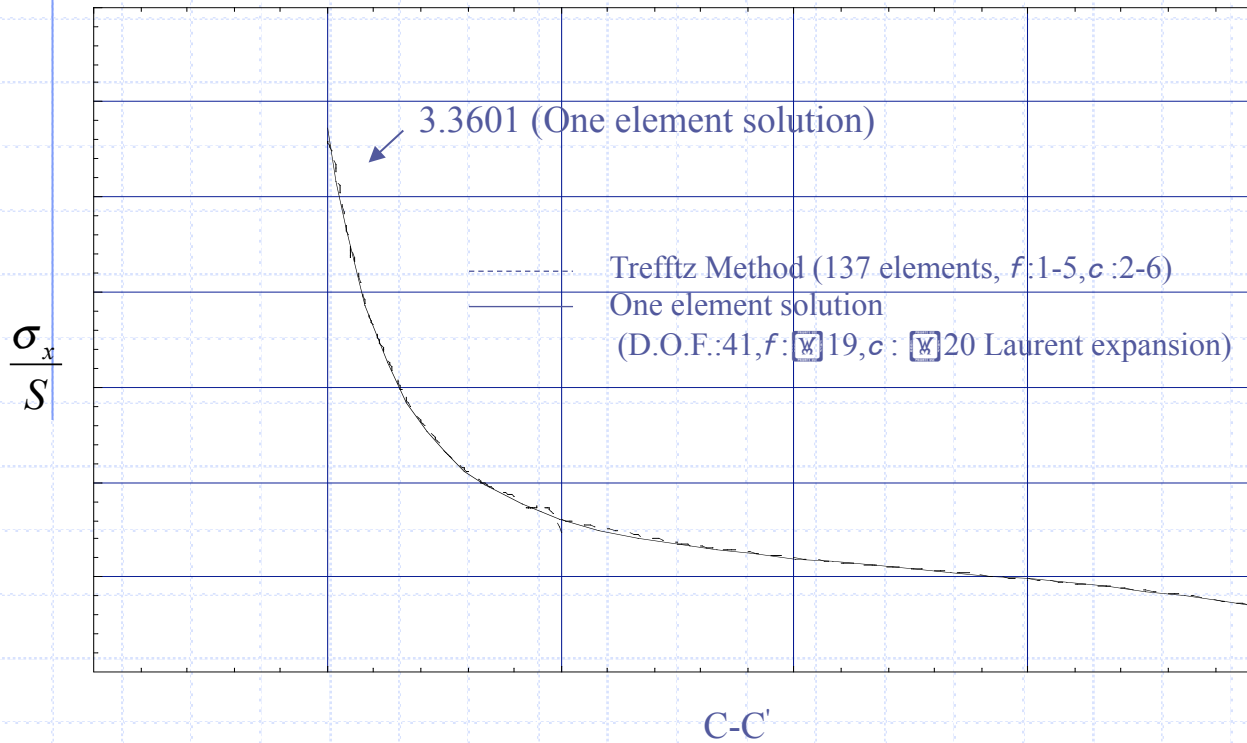
| DOF | TK1 | DOF | TK2 | DOF | TK3 | DOF | TK4 | DOF | TK5 | DOF | TK6 | DOF | TK7 |
|-----|-------|-----|-------|-----|-------|-----|-------|-----|-------|-----|-------|-----|-------|
| 10 | | 6 | 91.49 | 10 | 83.51 | 3 | 55.67 | 8 | | 4 | 90.82 | 20 | 93.75 |
| 18 | 77.22 | 12 | 80.16 | 16 | 92.89 | 7 | 77.42 | 12 | 78.16 | 6 | 87.81 | 28 | 87.50 |
| 28 | 80.30 | 20 | 86.12 | 24 | 81.52 | 13 | 90.72 | 16 | 87.28 | 8 | 82.74 | 40 | 86.21 |
| 40 | 83.91 | 30 | 86.55 | 34 | 88.32 | 21 | 82.92 | 20 | 83.44 | 10 | 87.46 | 52 | 85.84 |
| 54 | 87.40 | 42 | 85.56 | 46 | 84.45 | 31 | 87.67 | 24 | 91.11 | 12 | 85.39 | 64 | 85.72 |
| 70 | 86.48 | 56 | 86.12 | 60 | 86.62 | 43 | 84.88 | 28 | 85.86 | 14 | 86.05 | 76 | 85.70 |
| 88 | 85.78 | 72 | 85.78 | 76 | 85.18 | 57 | 86.50 | 32 | 86.74 | 16 | 85.89 | | |
| 108 | 85.29 | 90 | 85.97 | | | | | 36 | 86.07 | 18 | 85.83 | | |
| 154 | 86.01 | 110 | 85.87 | | | | | 40 | 86.00 | 20 | 85.95 | | |
| | | | | | | | | 60 | 85.94 | 30 | 85.93 | | |
| | | | | | | | | 80 | 85.88 | 40 | | | |



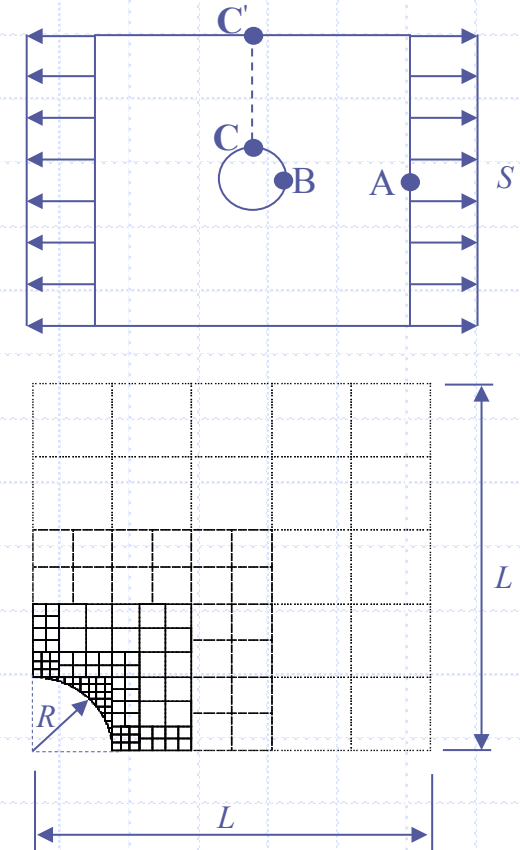
| DOF | Timoshenko |
|-----|------------|
| 1 | 82.99 |
| 3 | 86.18 |
| 5 | 86.10 |
| 7 | 86.10 |
| 9 | 86.10 |
| 11 | 86.10 |
| 17 | 86.10 |



一方向に一様分布引張荷重を受ける有孔正方形板の 応力集中解析(C-C'断面)



Trefftz法では右図のごとくサイズの異なる
メッシュ分割の重ね合わせが自由に行える。

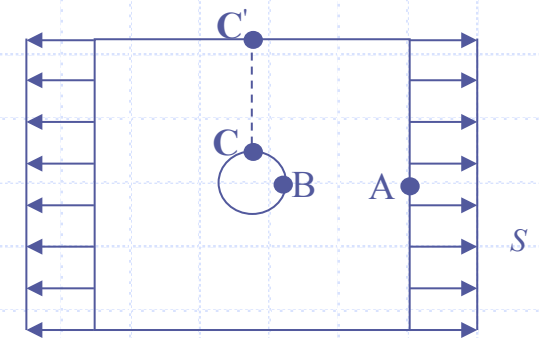


$$R = 10\text{mm}, L = 50\text{mm}, S = 100\text{kgf} / \text{mm}$$

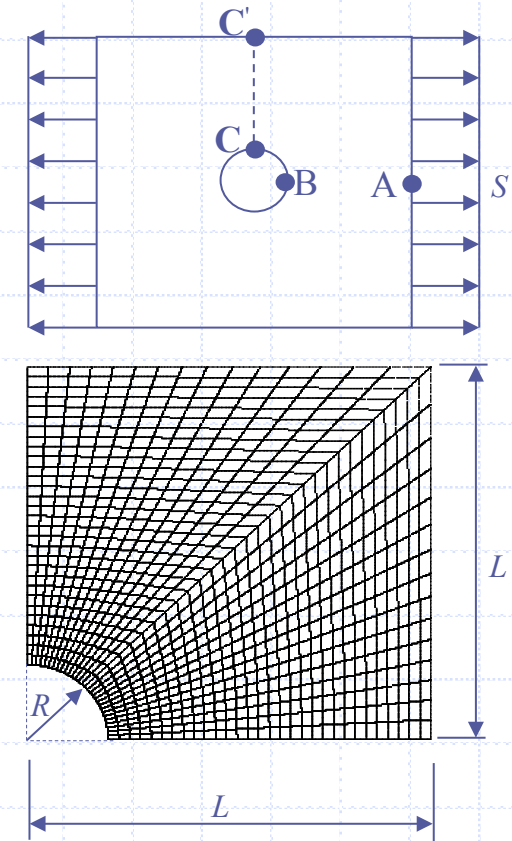
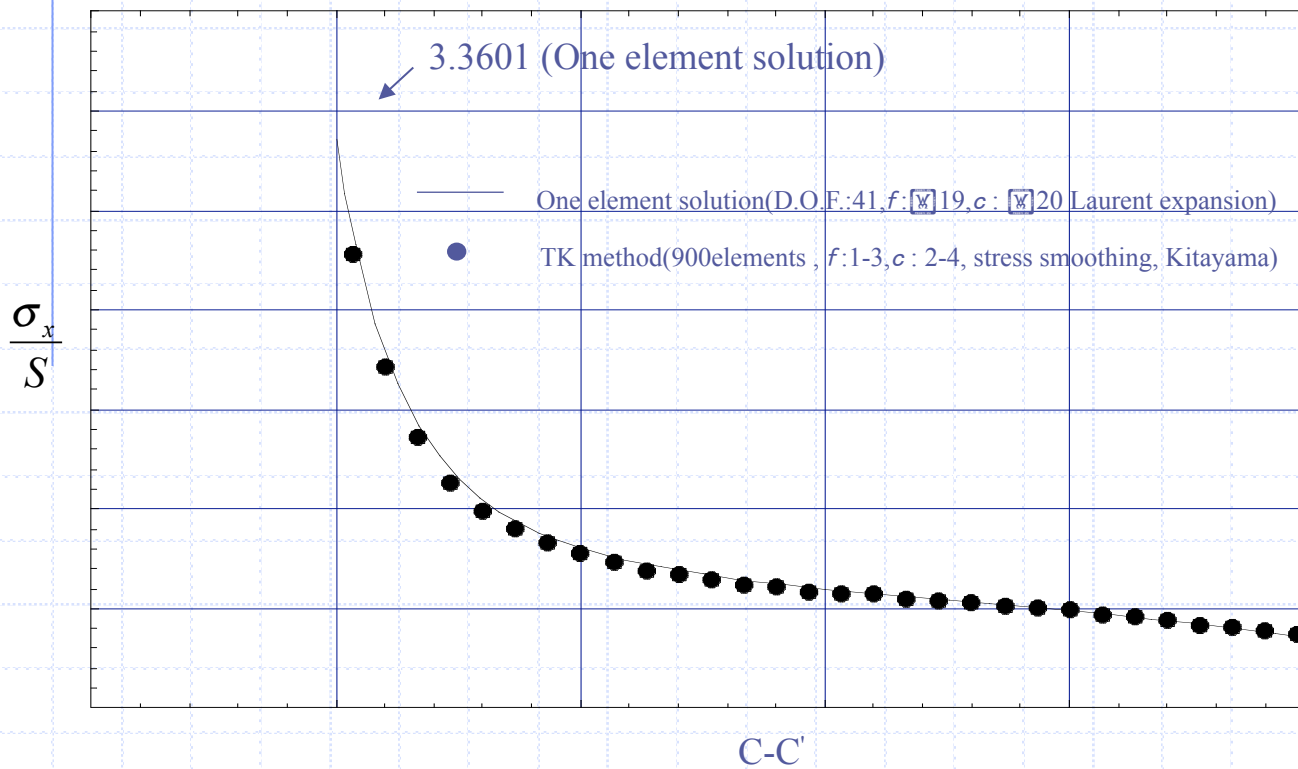
$$E = 20000\text{kgf} / \text{mm}^2, \nu = 0.3$$

一軸引張を受ける有孔正方形板の応力及び変位の計算値の収束性

| D.O.F | $(\sigma_x)_{\max}/S$ | u_A | u_B |
|-------|-----------------------|-----------|-----------|
| 17 | 3.351535 | 0.3048718 | 0.1682909 |
| 21 | 3.358187 | 0.3054907 | 0.1686762 |
| 25 | 3.359322 | 0.3061321 | 0.1687509 |
| 29 | 3.359656 | 0.3066006 | 0.1687975 |
| 33 | 3.360086 | 0.3064086 | 0.1687917 |
| 37 | 3.360099 | 0.3063427 | 0.1687925 |
| 41 | 3.360115 | 0.3064508 | 0.1687930 |



通常のメッシュ分割によるTK法の計算結果



$R = 10\text{mm}$, $L = 50\text{mm}$, $S = 100\text{kgf/mm}$
 $E = 20000\text{kgf/mm}^2$, $\nu = 0.3$